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Engineering Notes

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Spacecraft Euler Parameter Tracking of Large-Angle Maneuvers via Sliding Mode Control

Rush D. Robinett* and Gordon G. Parker†
Sandia National Laboratories, Albuquerque, New Mexico 87185

Introduction

SEVERAL papers have been written developing sliding mode control (SMC) for space applications. Vidali[1] showed the quadratic-regulator-based optimal sliding surface for a Euler parameter tracking satellite SMC controller. The typical set of four Euler parameters was truncated to a set of three independent Euler parameters. The three optimal sliding surfaces were shown to be functions of the three attitude Euler parameters and the body-fixed coordinate spacecraft angular velocities. The disturbance accommodation term of the control law was designed using constant valued parameters constrained by worst-case disturbance torque magnitudes. Simulation results were presented for linearized equations of motion. Nonlinear sliding surfaces, consisting of a combination of Rodrigues parameters and body-fixed angular rates, were examined for reorientation and detumbling maneuvers.[2,3] Again, the disturbance accommodation term of the control law was based on a worst-case estimate of the disturbance torque. Spacecraft control in the presence of model uncertainty was investigated for tracking Euler angle histories.[4] Although successful for some maneuvers, the control law relies on the inverse of a transformation matrix, which, for large angle reorientation maneuvers, becomes singular.

Here, an SMC method is presented for Euler parameter command tracking of spacecraft maneuvers. Although only three Euler parameters are independent, desired closed-loop performance is specified on all four Euler parameters by way of four sliding surfaces in the Euler parameter error-error rate phase planes. Euler parameter commands are tracked in a least squares sense while guaranteeing global asymptotic stability. Simulation results of a typical satellite configuration is presented.

Nonlinear Spacecraft Equations of Motion

Equations of motion for an arbitrary rigid spacecraft are derived from Newton's laws and Euler's equations. The spacecraft is maneuvered with three reaction wheels that are aligned with three mutually orthogonal axes.

Euler parameters are utilized to tolerate any orientation of the spacecraft without inherent singularities. The kinematic differential equations are

$$\dot{\beta} = \bar{\beta}\bar{\omega} \tag{1}$$

where β is the Euler parameter vector, $\beta = (\beta_0, \beta_1, \beta_2, \beta_3)^T$ subject to the constraint $\beta^T\beta = 1$; $\bar{\omega}$ is the augmented spacecraft body-fixed angular rate vector, $\bar{\omega} = (0, \omega_1, \omega_2, \omega_3)^T$; and $\bar{\beta}$ is the typical 4×4 orthogonal matrix of Euler parameters.

The spacecraft and reaction wheel equations are

$$(\bar{I} + J)\dot{\omega} + J\dot{\Omega} + \omega \times (\bar{I} + J)\omega + \omega \times J\Omega = M \tag{2}$$

$$T = J(\dot{\omega} + \dot{\Omega}) + C\Omega \tag{3}$$

respectively, where T is the reaction wheel torque vector, Ω is the vector of reaction wheel rates, C is the viscous friction damping matrix, and $\bar{I}$ and J are the spacecraft and reaction wheel inertia matrices, respectively.

Euler Parameter Tracking SMC Law

The first step in designing an Euler parameter tracking sliding mode controller is to derive a second-order representation of the nonlinear spacecraft equations of motion [i.e., Eq. (1)]. The generalized coordinates of choice are the Euler parameters, and since only three of the coordinates are independent, a modified form of Eq. (1) is used in the control design:

$$\dot{\beta} = D\omega \tag{4}$$

where D is a 4×3 matrix found by a suitable truncation of $\bar{\beta}$ (i.e., the first column of $\bar{\beta}$ is removed).

The time derivative of Eq. (4) provides the second-order representation,

$$\ddot{\beta} = \dot{D}\omega + D\dot{\omega} \tag{5}$$

and substitution of Eqs. (2) and (3) into Eq. (5) yields

$$\ddot{\beta} = \dot{D}\omega + D\bar{I}^{-1}(\tilde{N} - \tilde{T}) \tag{6}$$

where

$$\tilde{N} = M - \omega \times (\bar{I} + J)\omega - \omega \times J\Omega$$

and

$$\tilde{T} = T - C\Omega$$

The next step is to define the sliding surface for the Euler parameters. Four sliding surfaces are used in the control design, and the result is a least squares solution between the three torques and the four coordinates. The sliding surfaces are

$$s = W(\beta - \beta_r) + (\dot{\beta} - \dot{\beta}_r) = \mathbf{0} \tag{7}$$

with time derivative

$$\dot{s} = W(\dot{\beta} - \dot{\beta}_r) + (\ddot{\beta} - \ddot{\beta}_r) = \mathbf{0} \tag{8}$$

where W specifies the sliding surface dynamics. Upon substitution of Eq. (6), the control law for the reaction wheel torques becomes

$$T = \tilde{N} + \bar{I}D^*[-\ddot{\beta}_r + \dot{D}\omega + W(\dot{\beta} - \dot{\beta}_r)] + C\Omega - \Lambda\,\mathrm{sgn}(s) \tag{9}$$

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*Distinguished Member, Technical Staff. Senior Member AIAA.
†Senior Member, Technical Staff. Member AIAA.

where D^* is the pseudo or generalized inverse of D,

$$D^* = (D^T D)^{-1} D^T$$

$$(D^T D)^{-1} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

and Λ is a weight matrix that will be determined next.

The stability of the closed-loop system is determined by applying Lyapunov's direct method resulting in the constraint on the control design parameter Λ,

$$\Lambda = \bar{I} D^* A \tag{10}$$

The values of the W and A matrices of Eqs. (7) and (9) may be chosen by minimizing a quadratic performance function of the Euler parameter tracking error for a worst-case maneuver. This technique has been applied successfully for robotic applications using a recursive quadratic programming algorithm.[5]

An important feature of any closed-loop controller is its stability in the presence of disturbances or modeling errors. If the uncertainties are a result of spacecraft inertia uncertainty or bounded external disturbances, then further constraints can be placed on Λ, ensuring stability.

Simulation Results

The inertial properties of the satellite system considered are given in Table 1. The large-angle maneuver of interest is a simultaneous three-axis, Euler angle rotation of 60 deg accomplished in 20 s. Plots of Euler parameters and Euler parameter errors are shown in Figs. 1 and 2, where the tracking errors are maintained below 0.001. An inherent feature of the sliding mode controller is the ability to correct for initial condition errors. This is demonstrated by examining the Euler parameter response resulting from a perturbed initial condition in β_1. Figure 3 shows the time history of the first Euler parameter, clearly indicating the ability of the sliding mode controller to compensate for the initial condition error.

Table 1 Spacecraft inertial parameters

I_{11}, kgm^2	I_{22}, kgm^2	I_{33}, kgm^2	I_{12}, kgm^2	I_{13}, kgm^2	I_{23}, kgm^2	J_{11}, kgm^2	J_{22}, kgm^2	J_{33}, kgm^2
202	200	117	0.6	9.9	−7.3	0.1	0.1	0.1

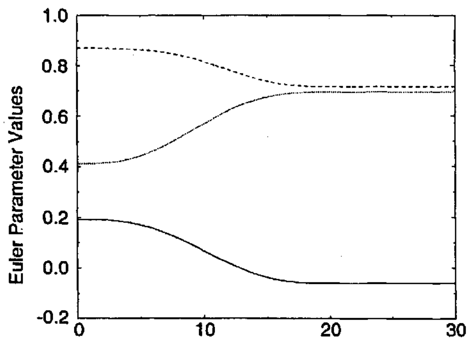

Fig. 1 Euler parameter histories for the nominal, 20-s maneuver: ——, β_1, β_3; ····, β_2; and - - - -, β_0.

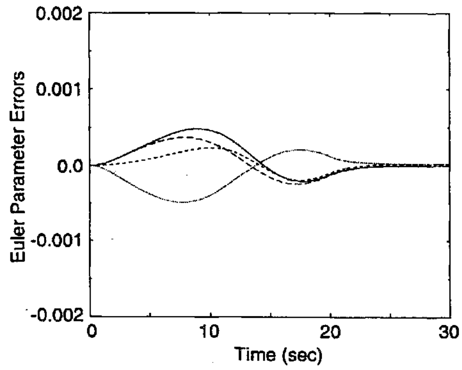

Fig. 2 Euler parameter error histories for the nominal, 20-s maneuver: ——, β_1; ····, β_2; – – –, β_3; and - - - -, β_0.

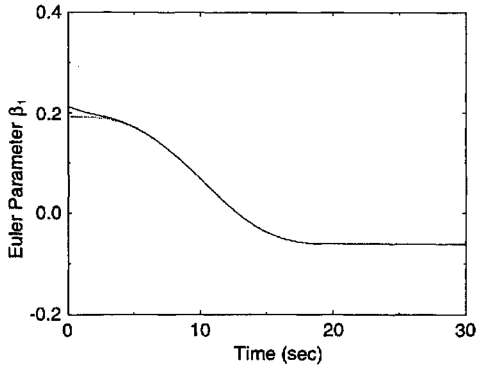

Fig. 3 Euler parameter histories of β_1 for the nominal maneuver with an initial condition error: ——, response and ····, command.

For all cases considered, the SMC gain matrices are chosen with $W = 10.0I$ and $\Lambda = 1.0I$, where I is the identity matrix. Furthermore, the sgn of the control law, Eq. (9), is replaced by a smoothed version implemented by

$$(\pi/2)a\tan(s)$$

to reduce the chattering phenomenon encountered during state motion along the sliding surface. SMC law stability has been shown, in general, not to be effected by such a modification, as long as the function has the same sign properties as the sgn.[6]

Summary

An Euler parameter SMC method was presented, which allows for explicit design of the four quaternion sliding surfaces. Euler parameters were chosen as the generalized coordinates for tracking to facilitate large angle spacecraft reorientation maneuvers. Since the four Euler parameter trajectories are not independent, a generalized inverse was used to solve for the momentum wheel torque inputs. By choosing the disturbance accommodation coefficient matrix in the SMC law as a function of the Euler parameters, a simple constraint was obtained for ensuring stability of the closed-loop system.

Acknowledgment

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